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The linear work hardening stage and de Broglie equation for autowaves of localized plasticity

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Abstract

The generality of localization of plastic deformation, which is observed at the stage of linear work hardening for HCP, BCC and FCC mono- and polycrystals of pure metals and alloys, is considered. It was found previously that the motion rate of localized flow autowave is related to the reciprocal value of the work hardening coefficient by a linear law, which is universal in character. This is further substantiated by the results of the given study. The waves of plastic flow localization are found to have dispersion law. It has been established that in order to address the autowave of localized deformation, a quasi-particle may be introduced. The quasi-particle's characteristics have been defined.

Keywords: Plastic flow; Linear work hardening; Autowave; Wavelength; Quasi-particle

1. Introduction

In recent years, intensive studies of localization of plastic flow have been under way. Thus it is found by Zuev and Danilov (1997, 1999), Zuev (2001), Zuev et al. (2001) that over the entire plastic flow curve $\sigma(\varepsilon)$, various forms of localization are realized in an orderly fashion. The shape of local strain pattern that emerges at a given stage of flow is in strict correspondence with the acting law of work hardening $G^{-1} \times d\sigma/d\varepsilon = \theta(\varepsilon)$. According to Zuev (2001), such forms of flow localization might be regarded as different kinds of autowaves. The most striking picture of localized plasticity is found to emerge at the stage of linear work hardening ($\theta = \text{const}$). In this instance, along the specimen axis there propagates a typical phase

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wave, which is characterized by constant values of wavelength λ, frequency ω and propagation velocity V_{aw}. It is found experimentally by Zuev and Danilov (1997, 1999) that the characteristic values of propagation velocity and wavelength are in the intervals of $10^{-5} \leqslant V_{aw} \leqslant 10^{-4}$ m/s and $5 \leqslant \lambda \leqslant 15$ mm, respectively. It has been established earlier by Zuev (2001), Zuev et al. (2001) that the propagation rate of autowaves of localized plastic deformation is inversely proportional to the work hardening coefficient, i.e. $V_{aw} \sim \theta^{-1}$.

Conceivably, the generation of this kind of wave processes is related (Haken, 1988) to a macro scale self-organization of elementary shears, which are involved in the plastic deformation of crystals. As the flow evolves from one stage to the next, in the deforming material there emerges a certain wave type corresponding to the respective stage of work hardening. In the works of Aifantis (1984, 1996a,b, 1999) the theoretical aspect of the problem of self-organization is considered and the likely shapes of local strain patterns, which were observed later on, are predicted.

In order to get an understanding of the nature of large-scale localization of plastic flow, data on wave process parameters and information about the manner in which the latter parameters are related to other characteristics of the materials investigated, especially during the deformation at the stage of linear work hardening, must be available. The present investigation was performed with a view of extending the propositions stated earlier to BCC single and polycrystals and HCP polycrystals and of establishing regularities which are common to all the investigated materials and which are associated with the origin of autowaves of plastic flow localization. From this standpoint the linear work hardening stage appears to be of specific interest since the picture of plastic deformation localization evolving in this case is a typical wave characterized by wavelength and propagation velocity.

2. Materials and experimental procedure

The wave rate data obtained previously for the linear stage of process are supplemented here by the addition of data, which were derived for specimens prepared from BCC single- and polycrystals of the Fe + 3 wt. %Si alloy and from FCC polycrystals of Al having grain size in the range of 10 μm to 10 mm. Listed in Table 1 are the data on the materials' structure and on the mechanisms involved in the deformation of all the investigated materials in which the linear work hardening stage is observable. In the case of FCC single crystals, the occurrence of the above flow stage has been unambiguously established; however, in the case of polycrystals, in particular Al, it is in no way a received fact, although there is compelling evidence for the existence of such a stage and a plausible explanation of likely realization of this law in aluminum reported by Jaoul (1957). The linear stage does exist on the plastic flow curve of polycrystalline

Table 1
The materials investigated (chemical composition in wt.%)

Metal or alloy	Symbols in Fig. 1	Lattice	Single or polycrystalline	Mechanism of deformation
Cu (pure)	+	FCC	Single	Dislocation
Ni (pure)	×	FCC	Single	Dislocation
Al (pure)	•	FCC	Poly-	Dislocation
Cu–10%Ni–6%Sn	*	FCC	Single	Dislocation
Fe–Cr–Ni–N	▲	FCC	Single	Dislocation
Fe–13%Mn–1%C	▼	FCC	Single	Dislocation
	▽	FCC	Single	Twinning
Fe–3%Si	♦	BCC	Single	Dislocation
Fe–3%Si	♦	BCC	Poly-	Dislocation
Zr–1%Nb	■	HCP	Poly-	Dislocation

aluminum. With growing grain size, however, this portion of the flow curve tends to diminish in length and at $D \geqslant 0.5$ mm disappears altogether, with the work hardening coefficient varying in the following manner: $\theta \sim D^{-1/2}$ (Zuev and Semukhin, 2002; Zuev et al., 2003). By choosing the appropriate composition and extension axis orientation, one can obtain different versions of the plastic flow curve for single γ-Fe base crystals and tailor the requisite conditions for deformation by twinning (Zuev et al., 2001). All the specimens were tested in tension; the movable grip of the test machine had motion rate $V_{\mathrm{mach}} = 1.67 \times 10^{-6}$ m/s. During the deformation of flat specimens tested in tension, the flow curve was plotted in the co-ordinates $\sigma - \varepsilon$ and the localization of deformation in the tensile specimens was recorded by the technique of speckle interferometry described by Jones and Wikes (1983) and Zuev et al. (2002). Using the data processing technique proposed by Zuev et al. (2001), one can define the wavelength λ (wave number $k = 2\pi/\lambda$) and period T (frequency $\omega = 2\pi/T$) and calculate the phase propagation rate $V_{\mathrm{aw}} = \lambda/T = \omega/k$ for the investigated waves.

3. Experimental results

Fig. 1 illustrates the dependence $V_{\mathrm{aw}}(\theta)$ derived from the data, which have been obtained for the materials tabulated in Table 1. Evidently, it has a hyperbolic form

$$V_{\mathrm{aw}} = \frac{\Xi}{\theta}, \tag{1}$$

where $\Xi = 6.33 \times 10^{-7}$ m/s. The correlation factor for the quantities V_{aw} and θ^{-1} is 0.95. Using a standard statistical treatment procedure (Hudson, 1964) the latter value was verified and the correlation obtained was found to be statistically significant. The regularity established herein is believed to be a universal one since among all the investigated materials whose flow curve shows the stage of linear work hardening, i.e. HCP, FCC and BCC single- and polycrystals of pure metals and alloys (those deforming by twinning included), there is not a single exception.

The fact that the wavelength and frequency can be experimentally obtained for the autowave process permitted establishment of dispersion law $\omega(k)$ (in dimensionless co-ordinates) for the same process (see Fig. 2a). This law has the quadratic form

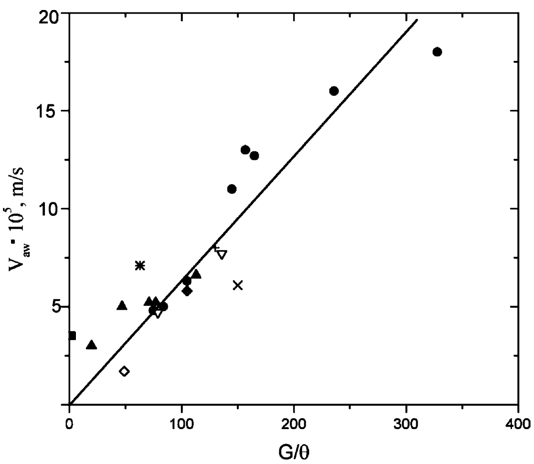

Fig. 1. The generalized dependence of the autowave propagation rate on the work hardening coefficient in the linear work hardening stages.

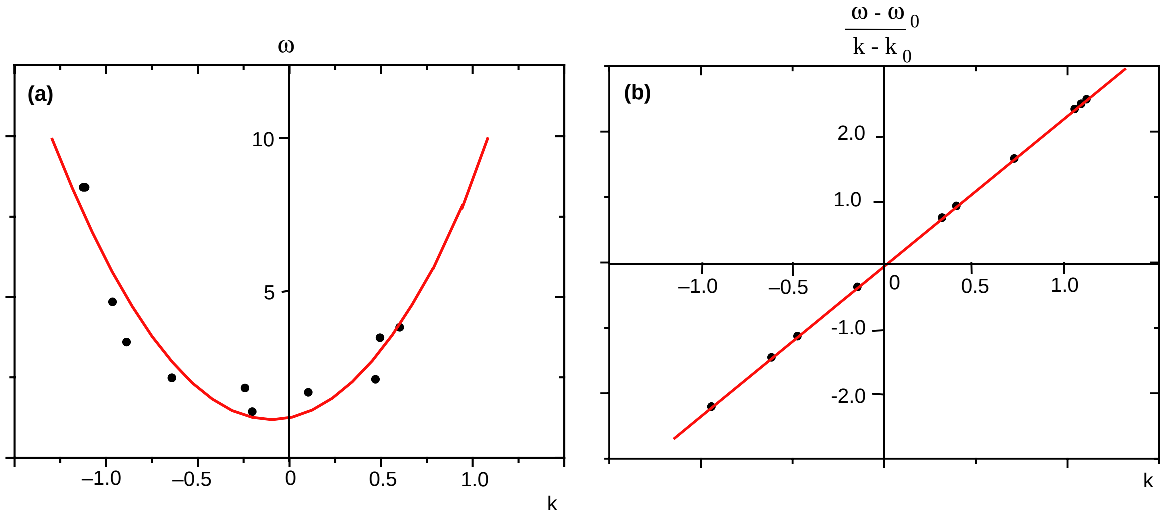

Fig. 2. (a) The generalized dispersion law of the autowaves in the linear work hardening stage (normalized values); (b) the same in the co-ordinates $\frac{\omega-\omega_0}{k-k_0} - k$.

$$\omega = 1 + k^2. \tag{2}$$

In Fig. 2b the above data are presented in the co-ordinates $\frac{\omega-\omega_0}{k-k_0} - k$ straightening the quadratic parabola (ω_0 and k_0 are the co-ordinates of a point in Fig. 2a); the linearity of the resultant plot supporting the validity of relation (2).

4. Discussion of results

First and foremost, it should be noted that the dependence of waves' motion velocity on the work hardening coefficient of the kind $V_{aw} \sim 1/\theta$ is different from the dependence, which has the form $V_{pw} \approx \sqrt{\theta/\rho} \sim \sqrt{\theta}$ and which was obtained for thoroughly investigated plasticity waves (see, for example, Kolsky, 1953; Davies, 1956). The difference in the functions $1/\theta$ and $\sqrt{\theta}$ suggests that the autowaves in question are fundamentally distinct from plasticity waves.

Now consider the possible reason for the existence of a dependence of the type $V_{aw} \sim 1/\theta$ by taking into account that on going from one substance to another, on changing extension axis orientation in single crystals or by varying grain size in polycrystals, the value of θ will change. In this instance, the assumption that $\mathrm{d}V_{aw}/\mathrm{d}\theta \sim \mathsf{L}(\varepsilon)$, ($\mathsf{L}$ is the glide path at the stage of linear work hardening) seems reasonable enough. Seeger (1957) holds that at the stage of linear work hardening $\mathsf{L} = \frac{\Lambda}{\varepsilon-\varepsilon^*}$, Λ is determined by the kind of material, while the deformation ε^* corresponds to the beginning of linear stage for which

$$\theta \sim \sqrt{\frac{Nb}{3\Lambda}}, \tag{3}$$

where N is the number of dislocations with the Buerger's vector b in a planar pileup. Correspondingly, for various materials $\mathrm{d}V_{aw}/\mathrm{d}\theta \sim \mathsf{L} \sim \Lambda \sim 1/\theta^2$. Hence

$$\frac{\mathrm{d}V_{aw}}{\mathrm{d}\theta} \sim \frac{Nb}{(\varepsilon - \varepsilon^*)\theta^2}. \tag{4}$$

The evidence reported by Seeger (1957) suggests that on going from one material to another, θ changes to a greater degree than $\varepsilon - \varepsilon^*$ does and in all cases, $N \leqslant 40$; therefore, in (4) one can set $\varepsilon - \varepsilon^* \approx \text{const}$, then from (4) follows $V_{aw} \sim 1/\theta$.

The data on the dispersion law of localized deformation autowaves (Fig. 2a and b) help refine our understanding on the origin of these waves. According to Dodd et al. (1982), relation (2) is characteristic of the so-called Schrödinger nonlinear equation used for the description of self-organization processes in nonlinear media. Evidently the oscillation spectrum has a gap $0 \leqslant \omega \leqslant \omega_{min}$. Using the initial dimensional ω and k values one can find that $\omega_{min} \approx 5 \times 10^{-2}\,\text{s}^{-1}$, so that $\hbar\omega_{min} \ll k_B T$ (k_B is the Boltzmann constant) and spontaneous localization of plastic flow is likely to occur at any temperature, provided geometric limitations imposed by small specimen size (Zuev, 2001) are absent.

The above dispersion relation (2) derived for autowaves of plastic flow localization corresponds formally with the dispersion law deduced for the de Broglie waves (see, for example, Crawford, 1973), which may have significant implications. Recently Billingsley (1998, 2001) applied the de Broglie equation to calculate mass by addressing autowaves; the calculated value was found to correlate with the atomic weight of the metal from which the specimen was made. In his work Billingsley (2001) identified the velocity of wave propagation, V_{aw}, in the de Broglie equation with the rate of tensile loading, V_{mach}, which is unjustifiable, since the experimental evidence reported by Zuev and Danilov (1997, 1999) suggests that the above values are related as $10 \leqslant V_{aw}/V_{mach} \leqslant 50$. Here the de Broglie equation is used in a more consistent manner by treating the experimental data, i.e., λ and V_{aw} values derived for the single Cu, Ni and γ-Fe crystals and polycrystalline Zr, V and Al.

Indeed, if one substitutes the experimental values of wavelength, λ, and of autowave propagation rate, V_{aw}, into the de Broglie equation $\lambda = h/mV$ (h is the Planck constant) as does Billingsley (2001) and makes use of the following relation to estimate mass

$$m = \frac{h}{\lambda V_{aw}}, \tag{5}$$

then m values can be calculated for the six metals investigated. The values of λ and V_{aw} are the averages obtained for all the test specimens made from a given metal or alloy. As is seen from Table 2, there is only an insignificant difference in the m values obtained; all the values have the scale $m_e \ll m \approx 1\,\text{amu}$ (m_e is the rest mass of electron; $1\,\text{amu} = 1.66 \times 10^{-27}\,\text{kg}$ is the atomic mass unit). The average mass calculated from (5) for the six metals investigated is $\langle m \rangle = (2.4 \pm 0.4) \times 10^{-27}\,\text{kg} = 1.43 \pm 0.23\,\text{amu} \approx 1.5\,\text{amu}$.

Further on, let us calculate the volume $\Omega = m/\rho$ by dividing m by the respective metal's density, ρ and a certain size $d_\Omega = \sqrt[3]{\Omega}$ (see Table 2). The values d_Ω are found to be close to the ion radii, r_{ion}, of the respective metals; in the case of Zr, V, Fe and Ni, the values d_Ω and r_{ion} are virtually the same, which is supported by the d_Ω/r_{ion} ratios listed in Table 2. The average obtained for the six elements investigated is $\langle d_\Omega/r_{ion} \rangle = 1.04$.

If one performs normalization of m values derived from (5) with respect to the atomic mass, M_{at}, of the respective metals to introduce dimensionless mass

Table 2
The microscopic characteristics of autowaves

Metal ($e/a \equiv n$)	$\lambda \times 10^3$ (m)	$V_{aw} \times 10^5$ (m/s)	$m \times 10^{27}$ (amu)	$\rho \times 10^{-3}$ (kg/m^3)	d_Ω (nm)	r_{ion} (nm)	d_Ω/r_{ion}	$s \times 10^2$
Cu (1)	4.5	8.0	1.84 (1.10)	8.9	0.059	0.072	0.82	1.74
Al (3)	7.2	11	0.84 (0.50)	2.7	0.068	0.051	1.33	1.87
Zr (4)	5.5	3.5	3.44 (2.07)	6.5	0.081	0.079	1.02	2.24
V (5)	4.0	7.0	2.37 (1.43)	6.1	0.073	0.074	0.99	2.79
Fe (8)	5.0	5.1	2.60 (1.57)	7.9	0.069	0.064	1.08	2.81
Ni (10)	3.5	6.0	3.16 (1.90)	9.9	0.068	0.069	0.99	3.24

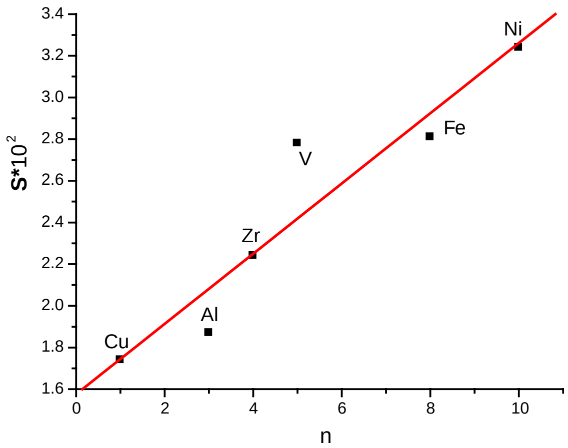

Fig. 3. The dependence of dimensionless mass, s, on electron concentration, $e/a \equiv n$, of the metal investigated.

$$s = \frac{m}{M_{\mathrm{at}}} \ll 1, \tag{6}$$

then in the range of metals investigated s grows linearly with increasing number of valence electrons per lattice unit (electron concentration, $e/a \equiv n$; see, for example, Cracknell and Wong, 1973) in the interval $1 \leqslant n \leqslant 10$, (see Fig. 3) i.e.

$$s = s_0 + \kappa \times n = 1.6 \times 10^{-2} + 0.17 \times 10^{-2} \times n. \tag{7}$$

The correlation factor for the quantities s and n is 0.95; according to Hudson (1964), it is statistically significant.

Finally, consider the estimation of mass associated with the existence of a gap in the oscillation spectrum (Fig. 2). Using the averaged velocity calculated for the six metals investigated, $\langle V_{\mathrm{aw}} \rangle = (6.8 \pm 1.3) \times 10^{-5}$ m/s, and setting $\omega_{\min} \approx 5 \times 10^{-2}\,\mathrm{s}^{-1}$ (Zuev, 2001), the average mass can be obtained from

$$\langle m \rangle^* = \frac{2\hbar\omega_{\min}}{\langle V_{\mathrm{aw}} \rangle^2} \approx (2.3 \pm 0.4) \times 10^{-27}\ \mathrm{kg} \approx (1.38 \pm 0.24)\ \mathrm{amu}. \tag{8}$$

The difference between the above two averages is statistically insignificant (Hudson, 1964).

5. Conclusion

The evidence presented and discussed herein has the following important implications. First and foremost, simple as they may seem, the estimates of the wave process parameters obtained from relation (5) are in no way trivial. They demonstrate that using a standard mathematical apparatus, macroscopic characteristics of the process of plastic flow localization, such as the wavelength and wave propagation rate of localized deformation, might be related to the characteristics of microscopic objects, which have

(i) effective mass, $0.5 \leqslant m \leqslant 2$ amu;
(ii) dimension (localization region), $d_{\Omega} \approx r_{\mathrm{ion}}$; and
(iii) velocity, $10^{-5} \leqslant V_{\mathrm{aw}} \leqslant 10^{-4}$ m/s.

This might imply that to the autowave processes of localized plastic flow there correspond certain quasi-particles. The quasi-particles' characteristics correlate with the respective parameters of the metals investigated, namely, quasi-particle size to ion radius, r_{ion}, and the mass cited above to concentration of electrons, $e/a \equiv n$.

In the second place, the proportionality $s \sim n$ derived appears to have considerable promise as a reliable footing for analysis of the impact exerted by the electron structure of metals and alloys on material strength and plasticity since variation of electron concentration $e/a \equiv n$ by alloying is expected to cause a change in the quantity $e/a \equiv n$ and in the related characteristics of plastic flow localization, such as wavelength (spacing between flow nuclei) and autowave propagation rate (Fig. 3).

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